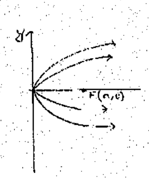
MATHEMATICS

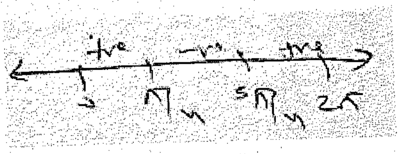
Class XII

COMMON PRE-BOARD EXAMINATION 2017-2018(SET-1)

MARKING SCHEME

Sr.no	ANSWER	MARK(S
	Section A	
1.	$\|A\|=5, so\ \|6A\|=6^3\|A\|=216 \times 5=1080$	1
2.	$y=e^{2\log(3x)}$ $y=9x^2$ $\frac{dy}{dx}=18x$	1
3.	$\|\vec{a}\|=1$, $\|\vec{b}\|=2$, $\vec{a}\cdot\vec{b}=1$ $\vec{a}\cdot\vec{b}=\|\vec{a}\|\,\|\vec{b}\|\cos\theta$ $1=1 \times 2\cos\theta$ $\cos\theta=\frac{1}{2}$ $\theta=60^0$ (or) $\frac{\pi}{3}$	1
4.	$\frac{d}{dx}[(\frac{dy}{dx})^3]=3(\frac{dy}{dx})^2 \times (\frac{d^2y}{dx^2})$ Order = 2, degree = 1 Sum = 2 + 1 =3	1
	Section B	
5.	$sin^{-1}\frac{3}{5}+sin^{-1}\frac{8}{17}$ $= tan^{-1}\frac{3}{4}+tan^{-1}\ 8/15$ $= tan^{-1}\left(\frac{\frac{3}{4}+\frac{8}{15}}{1-\frac{3}{4}\times\frac{8}{15}}\right)$ $= tan^{-1}\left(\frac{45+32}{60-24}\right)$ $= tan^{-1}\left(\frac{77}{36}\right)$ $= cos^{-1}\left(\frac{36}{85}\right)$	½ ½ 1
6.	**Y= f(x) =** $x^{\frac{1}{3}}$ Let x= 125, Δx= 2, $y=(125)^{1/3}=5$ $\frac{dy}{dx}=\frac{1}{3x^{\frac{2}{3}}}=\frac{1}{3(125)^{\frac{2}{3}}}=\frac{1}{75}$ dy = $\frac{1}{75}\times 2$ = 2/75 $\Delta y=0.26$	½ ½ 1

Q. No.	Solution	Marks
	Therefore $(127)^{\frac{1}{3}} = y + \Delta y \cong 5 + \Delta y \cong 5 + 0.26$ $\cong 5.026$	
7.	$I = \int_{-1}^{1} \log(2+3x)/(2-3x)dx$ $f(x) = log(\frac{2+3x}{2-3x})$ $f(-x) = \ log\left(\frac{2+3x}{2-3x}\right)^{-1}$ $= \ - log\left(\frac{2+3x}{2-3x}\right)$ $= - f(x)$ Hence, f is odd function So, $I = \int_{-1}^{1} \log(2+3x)/(2-3x)dx \ = 0$	1 1
8.	$4\sin^{-1}x + \cos^{-1}x = \pi$ $4\sin^{-1}x + \cos^{-1}x = 2(\sin^{-1}x) + 2\cos^{-1}x$ $2\sin^{-1}x = \cos^{-1}x$ $2\sin^{-1}x = \pi/2 - \sin^{-1}x$ $3\sin^{-1}x = \pi/2$ $\sin^{-1}x = \pi/6$ $x = \sin \pi/6$ $x = \frac{1}{2}$	½ ½ 1
9.	$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\2 & 1\end{bmatrix} = \begin{bmatrix}8 & 5\\20 & 13\end{bmatrix}$ I A = A $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -2\end{bmatrix} = \begin{bmatrix}8 & 5\\12 & 8\end{bmatrix}$ $R_2 \to R_2 - R_1$ A I = A $\begin{bmatrix}1 & 1\\3 & 1\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -1\end{bmatrix} = \begin{bmatrix}8 & 3\\12 & -4\end{bmatrix}$ $C_2 \to C_2 - C_1$	½ ½ 1
10.	$y^2 = 4ax$ -----> (1) $2y\frac{dy}{dx} = 4a$ ------>(2) From 1 & 2  $y^2 = 2y\left(\frac{dy}{dx}\right)x$ ⇨ $y^2 - 2y\frac{dy}{dx} = 0$ is required D.E / $\frac{dy}{dx} = \frac{y}{2x}$	½ ½ 1

11.	$\vec{a}\cdot\vec{b}=0$ => $\vec{a}\perp\vec{b}$ $\vec{a}\cdot\vec{c}=0$ => $\vec{a}\perp\vec{c}$ ⇨ $\vec{a} \parallel \vec{b}\times\vec{c}$ ⇨ $\vec{a}=\lambda(\vec{b}\times\vec{c})$---------->(1) ⇨ $1=\lvert\lambda\rvert\lvert\vec{b}\times\vec{c}\rvert$ ⇨ $1=\lvert\lambda\rvert\lvert\vec{b}\rvert\lvert\vec{c}\rvert\sin\frac{\pi}{6}$ ⇨ $1=\frac{1}{2}\lvert\lambda\rvert$ ⇨ $2=\lvert\lambda\rvert$ ⇨ $\lambda=\pm2$ ----> (2) ⇨ *so*, from 1 & 2 $\vec{a}=\pm2\,(\vec{b}\times\vec{c})$	½ ½ 1
12.	P (solved) = p(at least one solved) = 1 – p(none solved) = 1 – p($A'\cap B'$) = 1- P(A') X P(B') = $1-\frac{1}{2}\times\frac{2}{3}$ = $1-\frac{1}{3}=\frac{2}{3}$ (OR) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$ $=\frac{4}{6}=\frac{2}{3}$.	½ ½ 1
	SECTION – C	
13.	dx/dt = ap Cospt, dy/dt = -bp Sinpt dy/dx = $\frac{-bp\ Sinpt}{ap\ Cospt}$ = - b/a tanpt $\frac{d^2y}{dx^2}=\frac{-bp\ sec^2pt}{a}\times dt/dx$ $=\frac{-bp\ sec^2\ pt}{a}\times\frac{1}{pa\ cospt}=\frac{-b^2}{(a^2-x^2)y}$ ⇒ $(a^2-x^2)y\frac{d^2y}{dx^2}+b^2=0$.	1 1 1 1
14.	(x) = sinx + cosx F'(x) = cosx – sinx Now, f'(x) = 0 gives tanx = 1 => x = $\frac{\pi}{4}$, $\frac{5\pi}{4}$  	1 1 1 1

So, $f'(x) > 0$ for all $x \in [0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$

$F'(x) < 0$ for all $x \in (\frac{\pi}{4}, \frac{5\pi}{4})$

OR

Slope of the tangent to the given curve at Point (x,y) is given by

$$\frac{dy}{dx} = \frac{2}{(X-3)^2}$$ 1

⇨ $\frac{2}{(X-3)^2} = 2$ => x = 2,4 1

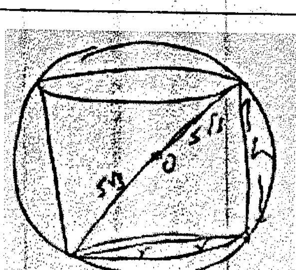
⇨ So the points are (2,2) and (4,-2) and the equation

⇨ y-2x+2=0 and y – 2x + 10 = 0

1

1

15.



$$h^2 + (2r)^2 = \left(10\sqrt{3}\right)^2$$

$$h^2 + 4r^2 = 300$$

$4h^2 = 300 - h^2$ ---------> (i)

Volume of cylinder = $\pi r^2 h$

$v(h) = \pi\left(\frac{3-h^2}{4}\right) \times h$ from equation (i) 1

$V(h) = \frac{\pi}{4}\ [300 - h^3]$

$V'[h] =\ [300 - 3h^2]$ 1

From critical point $v'[h] = 0$,

$3h^2 = 300$,

$h^2 = 100$

h = 10

so, $4r^2 = 300 - 10^2$

$r^2 = 50$ 1

$r = 5\sqrt{2}$

volume = $\pi r^2 h$

$\pi(50) \times 10 = 500\,\pi$ cubic unit

To check maxima 1

$V''[h] = \frac{\pi}{4}\ [-6h]$

$v''\ [h] = [-6 \times 10\ ]$

$= -15\,\pi < 0$

So, max volume = $500\,\pi$ cubic unit

16.

$\cos x = t \quad \Rightarrow -\sin x\, dx = dt$

$I = \int \frac{-dt}{(t^2+1)(t^2+4)}$

Put $t^2 = y$

$\frac{-1}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$

$-1 = A(y+4) + B(y+1)$

$(A+B) = 0$

$4A+B = -1$

$\Rightarrow \quad A = -1/3,\ B = 1/3$

This given integral $= -1/3 \int \frac{dt}{t^2+1} + 1/3 \int \frac{dt}{t^2+4}$

$= -1/3\, tan^{-1} t + 1/6\, tan^{-2}\left(\frac{t}{2}\right) + C$

$= -1/3\, tan^{-1}(Cos\ x) + 1/6\, tan^{-1}\left(\frac{Cos\ x}{2}\right) + c.$

Marks: 1, 1, 1, 1

17.

1) Applying $C_1 \to C1 + C_2 + C_3$

2) Taking 2 common from C_1

3) Applying $C_2 \to C_2 - C_1$

4) Applying $C_3 \to C_3 - C_1$

5) Applying $C_1 \to C_1 + C_2 + C_3$

6) Taking (-1) common from C_2 & C_3

Marks: 1, 1, 1, 1

18.

$$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

Since, f(x) is continuous at 0

So, L.H.L. = R.H.L. = f(0)=a

L.H.L, $\lim_{x\to 0^-} f(x)$

$= \lim_{x \to 0^-} a \sin\{{}^{\pi}/_{6}\ (x + 1)$

= a

R.H.L, $\lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3}$

$\lim_{x\to 0^+} \frac{\frac{\sin x}{cosx} - \sin x}{x^3}$

$\lim_{x\to 0^+} \frac{sinx\left(\frac{1-cosx}{\cos x}\right)}{x^3}$

$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{sin^2\ x/2}{x^2}$

$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{2\, sin^2\ x/2}{\frac{x^2}{4} \times 4}$

= 1 x 2/4 = 1/2

Marks: 1, 1, 1, 1

	So, a = ½ OR since f is differentiable at 1, f is continues at 1. Hence, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x+1$ $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (ax^2+b)$ = a + b f(1) = 3 As, f is continues at 1 , we have a + b = 3 ----(1) L.H.D. f'(1) $=\lim_{x\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{x\to 0} \frac{a(1-h)^2+b-3}{-h}$ $= \lim_{h\to 0} \frac{a+ah^2-2ah+b-3}{-h}$ $= \lim_{h\to 0} -ah+2a$ (from 1) = 2a R.H.D f'(1) $=\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{x\to 0} \frac{2(1+h)+1-3}{h}$ =2 As, f is differentiable at 1. We have 2a= 2, i.e. a = 1, b = 2.	
19.	$\frac{dy}{dx} - 3y\cot x = \sin 2x$ Here P= -3 cot x, Q= sin 2x I.f $= e^{\int p dx}$ $= e^{\int -3\cot x\, dx}$ $= e^{-3\log(\sin x)}$ $= e^{\log\left(\frac{1}{sin^3 x}\right)}$ $= \frac{1}{sin^3 x}$ $y \times I\cdot F = \int Q\cdot I\cdot F\, dx$ $= y\cdot\frac{1}{sin^3 x} = \int sin2x \times \frac{1}{sin^3 x} dx$ $= 2\int cotx\ cosecx\ dx + c$ $\frac{y}{sin^3 x}$ = -2 cosecx + c y = -2 sinx + $c sin^3 x$ 2 = -2 + c , C = 4 y = -2 sin^2x +4 $sin^3 x$ OR [x$sin^2(\frac{y}{x})$ - y]dx + x dy = 0 , $\frac{dy}{dx} = \frac{y-x\,sin^2\left(\frac{y}{x}\right)}{x}$ $\frac{dy}{dx} = \left(\frac{y}{x}\right) - sin^2\left(\frac{y}{x}\right)$, let $\frac{y}{x}$ = v => y = vx ⇨ $\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$ ⇨ $-\int \frac{dV}{sin^2 v} = \int \frac{dx}{x}$ ⇨ cot($\frac{y}{x}$) = log |x| + c	1 1 1 1 1 1

	⇨ $\cot(\frac{\pi}{4}) = \log(1) + c$, c = 1 ⇨ $\cot(\frac{y}{x}) = \log\lvert x\rvert + 1$ ⇨ OR ⇨ $\cot(\frac{y}{x}) = \log\lvert xe\rvert$	1 1
20.	$\vec{d} \perp \vec{a}$ and $\vec{d} + \vec{b}$ So, $\vec{d} \parallel \vec{a} \times \vec{b}$ ⇨ $\vec{d} = \lambda(\vec{a} \times \vec{b})$ --(1) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$ $\vec{a} \times \vec{b} = 32\hat{\imath} - \hat{\jmath} + (-14)\hat{k}$ ---(2) From (1) and (2) $\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$ But $\vec{d} \cdot \vec{c} = 15$ => $64\lambda + \lambda + 56\lambda = 15$ => $\lambda = 5/3$ So, $\vec{d} = \frac{5}{3}(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$ $= \frac{160}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{70}{3}\hat{k}$.	1 1 1 1
21.	$l_1: \frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $l_2: \frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$ If l_1 and l_2 are co-planner then, ⇨ $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ ⇨ $\begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$ ⇨ $\begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$ as R_1 and R_3 are identical. Hence the equation of plane $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ ⇨ $\begin{vmatrix} x-1 & y-3 & z \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$ ⇨ (x-1)(4-2)-(y-3)(2+3)+ z(-4 – 12) = 0 ⇨ 2(x-1) – 5(y-3) – 16z = 0	1 1 1

⇨ 2x – 5y -16z +2 +15 =0

⇨ 2x – 5y – 16z= - 17

⇨ $\vec{r}\cdot(2\hat{i}-5\hat{j}-16\hat{k}) = -17.$ 1

22. Let x denote the random variable and can take value 0,1,2, n=2, P=1/4, q= ¾ 1

x_i	0	1	2	Total	Marks
P_i	${}^2C_0\left(\frac{3}{4}\right)^2=\frac{9}{16}$	${}^2C_1\frac{1}{4}\left(\frac{3}{4}\right)=\frac{6}{16}$	${}^2C_2\left(\frac{1}{4}\right)^2=\frac{1}{16}$		[1+1/2]
x_iP_i	0	6/16	2/16	1/2	
x_i^2	0	6/16	4/16	5/8	[1/2]

1

Mean = $\sum xi\ pi = 1/2$ 1

Value = $\sum xi^2\, p(xi) - (\sum xi\ pxi)^2$

= 5/8 – 1/4 = 3/8. 1

23. Let E1,E2 and A are the event defined below

E1 = the missing card in a heart card

E2 = the missing card is not heart

A = drawing two heart card from the remaining cards 1

P(E1) =1/4 P(E)= 3/4

P(A/E1)= $\frac{12c_2}{51c_2}$ P(A/E2)= $\frac{13c_2}{51c_2}$ 1

By Bayes theorem

P(E1/A)= $\frac{P(E1).P(A/E1)}{P(E1).P\left(\frac{A}{E1}\right).P(E2).P(A/E2)}$

$\frac{\frac{1}{4}\times\frac{12c_2}{51c_2}}{\frac{1}{4}\times\frac{12c_2}{51c_2}+\frac{3}{4}.\frac{13c_2}{51c_2}}$ = 11/50 1

1

SECTION – D

24. $= \begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ B= $\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$

AB = $\begin{bmatrix}2+4+0 & 2-2 & -4+4\\ 4-12+8 & 4+6-4 & -8-12+20\\ 0-4+4 & 0+2-2 & 0-4+10\end{bmatrix}$ 2

$= \begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$

AB= 6I

Ax($\frac{1}{6}$B) = I 2

A^{-1} = 1/6(B) = 1/6 $\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ 2

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

AX= C

$X = A^{-1} C$

$X = 1/6 \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$X = 1/6 \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$ 2

$= 1/6 \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

X = 2, y= -1, Z = 4

OR

$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 2

⇨ $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ $R_2 \to R_2 + R_1$ 2

⇨ $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} A$ $R_2 \to R_2 + 2R_3$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 + 2R_2$

⇨ $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -4 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$

⇨ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$

⇨ Hence $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

25.	A	R= { (a,b): a,b ∈ R and $a - b + \sqrt{3} \in S$ S = Irrational Reflexive : Let a ∈ R so $a - a + \sqrt{3} \in S$ Hence, ∀ a ∈ R (a,a) ∈ R so R is reflexive Symmetric : Let $a = \sqrt{3}$ b = 2 So, $a - b + \sqrt{3} = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} - 2 \in S$, so (a,b) ∈ S Now, $b - a + \sqrt{3} = 2 + \sqrt{3} - \sqrt{3} = 2 \notin S$ .so, (b,a) ∉ R So, (a,b) ∈ R but (b,a) ∉ R so, R is not symmetry Transitive: Let $a = \sqrt{3}$, b = 2 $c = 2\sqrt{3}$ So, $b - a + \sqrt{3} = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} - 2 \in S$ So, (a,b) ∈ R Now, $b - c + \sqrt{3} = 2 - 2\sqrt{3} + \sqrt{3} = 2 - \sqrt{3} \in S$ (b,c) ∈ R But, $a - c + \sqrt{3} = \sqrt{3} - 2\sqrt{3} + \sqrt{3} = 0 \notin S$ So, (a,c) ∉ R hence, R is not a transitive	2 2
	NO-24. (OR)	$f(x) = 4x^2 + 12x + 15$ One- 0ne : For any x1 and x2 ∈ N we find that f(x1) = f(x2) $4x_1 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ ⇨ (x1 – x2) = 0 ⇨ X1 = x2 so, f : N -> Range (f) is one to one Since f : N -> Range (f) so, codomain = Range Hence f is onto , hence f : N -> range (f) is invertible Let f^{-1} denotes the inverse of f then f o $f^{-1}(x) = x$ for all x ∈ Range (f), $f(f^{-1}(x)) = x$ $4\{f^{-1}(x)\}^2 + 12 f^{-1}(x) + 15 = x$ $= f^{-1}(x) = \frac{-12 \pm \sqrt{144 - 16(15 - x)}}{8}$ $= f^{-1}(x) = \frac{-3 + \sqrt{x - 6}}{2}$ [∴ $f^{-1}(x) \in N$ ➔ $f^{-1}(x) > 0$]	2 2 2

		2

26.

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$= \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2 (\pi - x) + b^2\sin^2(\pi - x)}$$

$$= \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \quad -I$$ 2

$$2I = \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$2I/2 = 2\pi/2\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$ 2

$$I = \pi \int_0^{\infty} \frac{dt}{a^2 + b^2 + 2} \qquad \text{where } t = \tan x$$

$$I = \frac{\pi}{b^2}\int_0^{\infty} \frac{dt}{(\frac{a}{b})^2 + t^2}$$ 2

$$I = \frac{\pi}{b^2}\{b/a \tan^{-1}[\frac{bt}{a}]_0^{\infty}\}$$

$$= \frac{\pi}{b^2}\{b/a \times [\pi/2 - 0]\}$$

$$= \frac{\pi^2}{2ab}.$$ 2

OR

$$\int_0^1 e^{2-3x}\,dx$$

$$I = \int_0^1 e^{2-3x}\,dx$$

$$= \lim_{h\to 0} h\{f(0) + f(h) + f(2h) \dots + f(n-1)h\}$$

$$= \lim_{h\to 0} h[e^2 + e^{2-3h} + \cdots e^{2-3(n-1)}h]$$

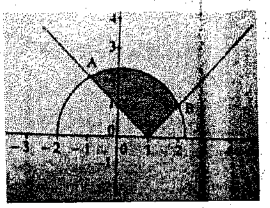
$$= \lim_{h\to 0} he^2[1 + e^{-3h} + \cdots + e^{-3(n-1)}h]$$ 2

$$= \lim_{h\to 0} he^2\,[\frac{(e^{-3nh} - 1}{e^{-3h} - 1}]$$

$$= \lim_{h\to 0} \frac{e^{-3} - 1}{\frac{e^{-3h} - 1}{-3h}}\,X\left(-\frac{1}{3}\right) = e^2(e^{-3} - 1)\,X\,(-\frac{1}{3})$$ 2

$$= \frac{1}{3}(e^2 - e^{-1})$$

27. 2



Solving $y = \sqrt{5 - x^2}$, $y = |x - 1|$

We get $(x - 1)^2 = 5 - x^2$

⇨ x = 2 and x = -1 2

the required area $= \int_{-1}^{1}\{\sqrt{5 - x^2} - (1 - x)\}\,dx + \int_1^2\{\sqrt{5 - x^2} - (x - 1)\}\,dx$

$= \int_{-1}^{2}\sqrt{5 - x^2}\,dx - \int_{-1}^{1}(1 - x)\,dx + \int_1^2 -(x - 1)\,dx$ 2

$= \frac{1}{2}\,[x\sqrt{5 - x^2} + 5\sin^{-1}\frac{x}{\sqrt{5}}]^2 - [x - \frac{x^2}{2}]_{-1}^{1} - [\frac{x^2}{2} - x]_1^2$

$= [-\frac{1}{2} + \frac{5}{2}(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}}]_{-1}^{2} - [x - \frac{x^2}{2}]_{-1}^{1} - [\frac{x^2}{2} - x]_1^2$

$= \left[-\frac{1}{2} + \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}}\right)\right]$ sq. units.

28.

$$\vec{n} = \overrightarrow{b_1} \, X \, \overrightarrow{b_2} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 2 & -1 & -3 \end{bmatrix} = 5\hat{\imath} + 7\hat{\jmath} + \hat{k}$$ 2

The equation of the planes $\vec{r} \cdot (5\hat{\imath} + 7\hat{\jmath} + \hat{k}) = 1$

The position vectors of any point on the given line is

$(1 + \lambda)\hat{\imath} + (2 + 3\lambda)\hat{\jmath} + (-1 - 9\lambda)\hat{k}$ we have

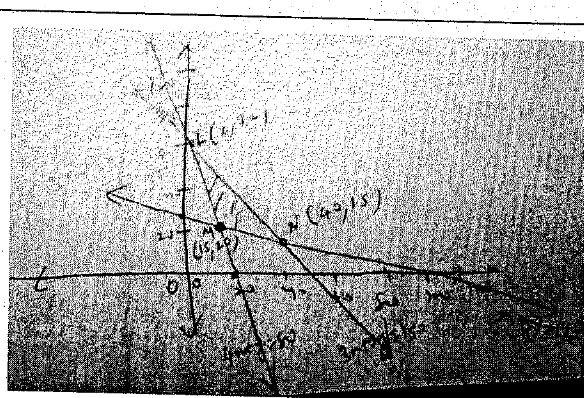
$$(1 + \lambda)5 + (2 + 3\lambda)7 + (-1 - 9\lambda)(1) = 1$$ 2

⇨ $\lambda = -1$.

⇨ The position vector of the required point is $-\hat{\jmath} + 8\,\hat{k}$

2

29.



2

Let x and y be the number of packets of food P and Q respectively . obviously 2

$$x \geq 0 \, , y \geq 0$$

Minimize z = 6x + 3y (vitamin A)

subject to constraints

12x+3y ≥ 240 (constraint on calcium)

4x- 20 y ≥ 460 (constraint on iron)

6x + 4y ≤ 300 (constraint on cholestrol)

$$x \geq 0 \, , y \geq 0$$

The coordinates of the corner part L,M and N

Corner point	Z = 6x + 3y
(2,72)	228
(15,20)	150 minimum
(40,15)	285

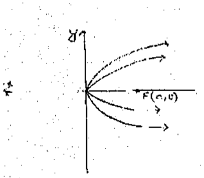
So z is minimum at the point (15,20).

So vitamin A amount will be minimum if 15 packet of food P and 20 packet 0f food Q are used in the special diet the minimum amount of vitamin A will be 150 units. 2

MATHEMATICS

Class XII

S.no	ANSWER	MARK(S)
	COMMON PRE-BOARD EXAMINATION 2017-2018(SET-2) MARKING SCHEME	
	Section A	
1.	$\|\vec{a}\| = 1$, $\|\vec{b}\| = 2$, $\vec{a}\cdot\vec{b} = 1$ $\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta$ $1 = 1 \times 2\cos\theta$ $\text{Cos}\theta = \frac{1}{2}$ $\theta = 60^0$	1
2.	$\frac{d}{dx}[(\frac{dy}{dx})^3] = 3(\frac{dy}{dx})^2 \times (\frac{d^2y}{dx^2})$ Order = 2, degree = 1 Sum = 2 + 1 =3	1
3.	$\|A\| = 5, so\ \|6A\| = 6^3\|A\| = 216 \times 5 = 1080$	1
4.	$y = e^{2\log(3x)}$ $y = 9x^2$ $\frac{dy}{dx} = 18x$	1
	Section B	
5.	$sin^{-1}\frac{3}{5} + sin^{-1}\frac{8}{17}$ $= tan^{-1}\frac{3}{4} + tan^{-1}\ 8/15$ $= tan^{-1}\left(\frac{\frac{3}{4}+\frac{8}{15}}{1-\frac{3}{4}\times\frac{8}{15}}\right)$ $= tan^{-1}\left(\frac{45+32}{60-24}\right)$ $= tan^{-1}\left(\frac{77}{36}\right)$ $= cos^{-1}\left(\frac{36}{85}\right)$	½ ½ 1
6.	$4\sin^{-1}x + \cos^{-1}x = \pi$ $4\sin^{-1}x + \cos^{-1}x = 2(\sin^{-1}x) + 2\cos^{-1}x$ $2\sin^{-1}x = \cos^{-1}x$ $2\sin^{-1}x = \pi/2 - \sin^{-1}x$ $3\sin^{-1}x = \pi/2$ $\sin^{-1}x = \pi/6$ $x = \sin \pi/6$	½ ½ 1

	$x = \frac{1}{2}$	
7.	$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\2 & 1\end{bmatrix} = \begin{bmatrix}8 & 5\\20 & 13\end{bmatrix}$ Use form IA = A then AI = A $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -2\end{bmatrix} = \begin{bmatrix}8 & 5\\12 & 8\end{bmatrix}$ $R_2 \to R_2 - R_1$ $\begin{bmatrix}1 & 1\\3 & 1\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -1\end{bmatrix} = \begin{bmatrix}8 & -3\\12 & -4\end{bmatrix}$ $C_2 \to C_2 - C_1$	1 1
8.	• $Y = f(x) = x^{\frac{1}{3}}$ Let x= 125, Δx= 2, $y = (125)^{1/3} = 5$ $\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3(125)^{\frac{2}{3}}} = \frac{1}{75}$ dy = $\frac{1}{75} \times 2 = 2/75$ $\Delta y = 0.26$ Therefore $(127)^{\frac{1}{3}} = y + \Delta y = 5 + \Delta y = 5 + 0.26$ = 5.026	½ ½ 1
9.	• $I = \int_{-1}^{1} \log(2+3x)/(2-3x)dx$ $f(x) = log(\frac{2+3x}{2-3x})$ $f(-x) = log\left(\frac{2+3x}{2-3x}\right)^{-1}$ $= - log\left(\frac{2+3x}{2-3x}\right)^{-1}$ = - f(x) Hence, f is odd function So, I= $\int_{-1}^{1} \log(2+3x)/(2-3x)dx = 0$	½ ½ 1
10.	• $y^2 = 4ax$ -----> (1) $2y\frac{dy}{dx} = 4a$ ------>(2) From 1 & 2 $y^2 = 2y\ (\frac{dy}{dx})x$ 	½ ½

		$\Rightarrow y^2 - 2y\frac{dy}{dx} = 0$ is required D.E	1
1.	$\vec{a}\cdot\vec{b}=0 \;=> \vec{a}\perp\vec{b}$	$\vec{a}\cdot\vec{c}=0 \;=> \vec{a}\perp\vec{c}$ $\Rightarrow \vec{a}\parallel\vec{b}\times\vec{c}$ $\Rightarrow \vec{a}=\lambda(\vec{b}\times\vec{c})$-------->(1) $\Rightarrow 1=\lvert\lambda\rvert\lvert\vec{b}\times\vec{c}\rvert$ $\Rightarrow 1=\lvert\lambda\rvert\lvert\vec{b}\rvert\lvert\vec{c}\rvert\sin\frac{\pi}{6}$ $\Rightarrow 1=\frac{1}{2}\lvert\lambda\rvert$ $\Rightarrow 2=\lvert\lambda\rvert$ $\Rightarrow \lambda=\pm 2$ ----> (2) $\Rightarrow$ *so*, from 1 & 2 $\vec{a}=\pm 2\,(\vec{b}\times\vec{c})$	½ ½ 1
2.		P (solved) = p(at least one solved) = 1 – p(none solved) = 1 – p($A'\cap B'$) = 1- P(A') X P(B') $= 1 - \frac{1}{2}\times\frac{2}{3}$ $= 1 - \frac{1}{3} = \frac{2}{3}$	½ ½ 1
		SECTION – C	
3.		1) Applying $C_1 \rightarrow C_2 + C_3$ 2) Taking 2 common from C_1 3) Applying $C_2 \rightarrow C_2 - C_1$ 4) Applying $C_3 \rightarrow C_3 + C_1$ 5) Applying $C_1 \rightarrow C_1 + C_2 + C_3$ 6) Taking (-1) common from C_2 & C_3	1 1 1 1
14.		$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0 \\ \frac{\tan x-\sin x}{x^3}, & x\rangle 0\end{cases}$ Since, f(x) is continuous at 0 So, L.H.L. = R.H.L. = f(0)=a L.H.L, $\lim_{x\to 0^-}$ f(x) $= \lim_{x\to 0^-} a\sin\{\pi/6\,(x+1)$ = a	1 1

R.H.L, $\lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3}$

$\lim_{x\to 0^+} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$

$\lim_{x\to 0^+} \frac{\sin x \left(\frac{1-\cos x}{\cos x}\right)}{x^3}$

$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{\sin^2 x/2}{x^2}$

$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{2\sin^2 x/2}{\frac{x^2}{4} \times 4}$ 1

= 1 x 2/4 = 1/2 1

So, a = ½

OR

since f is differentiable at 1, f is continues at 1.

Hence, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x + 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (ax^2 + b)$ = a + b

f(1) = 3

As, f is continues at 1 , we have a + b = 3 ----(1)

L.H.D. $f'(1) = \lim_{x\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{x\to 0} \frac{a(1-h)^2+b-3}{-h}$ 1

$= \lim_{h\to 0} \frac{a+ah^2-2ah+b-3}{-h}$

$= \lim_{h\to 0} -ah + 2a$ (from 1) 1

= 2a

R.H.D $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{x\to 0} \frac{2(1+h)+1-3}{h}$

=2 1

As, f is differentiable at 1. We have 2a= 2, i.e. a = 1, b = 2.

1

15. 15 $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$,

$\frac{dx}{dt} = ta\cos t, \quad \frac{dy}{dt} = at\sin t$ 1

$\frac{dy}{dx} = \tan t$ 1

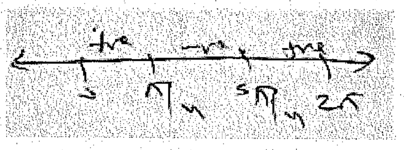
$\frac{d^2y}{dx^2} = sec^2\, t\, dt/dx$ 1

$= sec^2 t\, X \frac{1}{ta\cos t} = \frac{1}{ta} sec^3 t.$ 1

16. . F(x) = sinx + cosx 1

F'(x) = cosx – sinx

Now, f'(x) = 0 gives tanx = 1 => x = $\frac{\pi}{4}$, $\frac{5\pi}{4}$ 1



1

So, f'(x)> 0 for all x $\in [0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$ 1

F'(x)<0 for all x $\in (\frac{\pi}{4}, \frac{5\pi}{4})$

OR

Slope of the tangent to the given curve at Point (x,y) is given by 1

$$\frac{dy}{dx} = \frac{2}{(X-3)^2}$$ 1

⇨ $\frac{2}{(X-3)^2} = 2$ => x = 2,4

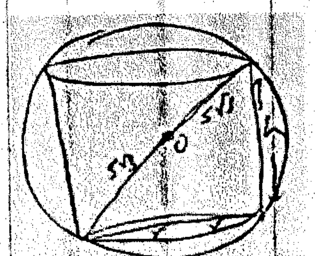
⇨ So the points are (2,2) and (4,-2) and the equation 1

⇨ y-2x+2=0 and y – 2x + 10 = 0

1

17.

1



$$h^2 + (2r)^2 = \left(10\sqrt{3}\right)^2$$

$$h^2 + 4r^2 = 300$$

$4h^2 = 300 - h^2$ ---------> (i)

Volume of cylinder = $\pi r^2 h$

$v(h) = \pi\left(\frac{3-h^2}{4}\right) \times h$ from equation (i)

$V(h) = \frac{\pi}{4}\ [300 - h^3]$

$V'[h] = [300 - 3h^2]$

From critical point $v'[h] = 0$, 1

$3h^2 = 300$,

$h^2 = 100$

h = 10

so, $4r^2 = 300 - 10^2$

$r^2 = 50$

$r = 5\sqrt{2}$

volume = $\pi r^2 h$ 1

$\pi(50) \times 10 = 500\,\pi$ cubic unit

To check maxima

$V''[h] = \frac{\pi}{4}\,[-6h]$

$v''[h] = [-6 \times 10]$

$= -15\,\pi < 0$

So, max volume = $500\,\pi$ cubic unit 1

18.	$I = \int \frac{dx}{x(x^4-1)}dx = \int \frac{x^3 dx}{x^4(x^4-1)}$	1
	Let $x^4 = y$, $4x^3$ dx = dy	1
	x^3dx $= \frac{1}{4}dy$, $I = \frac{1}{4}\int \frac{dy}{y(y-1)} = \frac{1}{4}[\int \frac{dy}{y-1}] - \int \frac{dy}{y}]$	
	$= \frac{1}{4}[log \frac{(y-1)}{y}] + c$	1
	$I = \frac{1}{4}[log \frac{(x^4-1)}{x^4}] + c$	1
19.	$\frac{dy}{dx} - 3y\cot x = \sin 2x$	
	Here P= -3 cot x, Q= sin 2x	
	I.f $= e^{\int pdx}$	
	$= e^{\int -3\cot x\, dx}$	
	$= e^{-3\log(\sin x)}$	
	$= e^{\log\left(\frac{1}{sin^3 x}\right)}$	1
	$= \frac{1}{sin^3 x}$	
	$y \times I \cdot F = \int Q \cdot I \cdot F\, dx$	1
	$= \quad y \cdot \frac{1}{sin^3 x} = \int sin2x \times \frac{1}{sin^3 x} dx$	
	$= \; 2\int cotx\ cosecx\ dx + c$	1
	$\frac{y}{sin^3 x}$ = -2 cosecx + c	
	y = -2 sinx + $c sin^3 x$	1
	2 = -2 + c , C = 4	
	y = -2 sin^2x +4 $sin^3 x$	
	NO-19. OR	
	[$x sin^2(\frac{y}{x})$ - y]dx + x dy = 0 , $\frac{dy}{dx} = \frac{y - x\, sin^2\left(\frac{Y}{x}\right)}{x}$	1
	$\frac{dy}{dx} = \left(\frac{Y}{x}\right) - sin^2\left(\frac{Y}{x}\right)$, let $\frac{Y}{x}$ = v => y = vx	1
	⇨ $\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$	
	⇨ $-\int \frac{dV}{sin^2 v} = \int \frac{dx}{x}$	
	⇨ cot$(\frac{Y}{x})$ = log |x| + c	
	⇨ cot$(\frac{\pi}{4})$ = log (1) +c, c = 1	1
	⇨ cot$(\frac{Y}{x})$ = log |x| + 1	
	⇨ OR	1

	$\Rightarrow \cot(\frac{y}{x}) = \log\|xe\|$	
20.	$\vec{d} \perp \vec{a}$ and $\vec{d} + \vec{b}$	1
	So, $\vec{d} \parallel \vec{a} X \vec{b}$	
	$\Rightarrow \vec{d} = \lambda(\vec{a} X \vec{b})$ --(1)	1
	$\vec{a} X \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$	
	$\vec{a} X \vec{b} = 32\hat{\imath} - \hat{\jmath} + (-14)\hat{k}$ ---(2)	
	From (1) and (2)	
	$\vec{d} = \lambda(\vec{a} X \vec{b}) = \lambda(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$	
	But $\vec{d} \cdot \vec{c} = 15$	1
	$\Rightarrow 64\lambda + \lambda + 56\lambda = 15$	
	$\Rightarrow \lambda = 5/3$	
	So, $\vec{d} = \frac{5}{3}(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$	
	$= \frac{160}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{70}{3}\hat{k}$.	1
21.	$l_1: \frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $l_2: \frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$	1
	If l_1 and l_2 are co-planner then,	
	$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$	
	$\Rightarrow \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$	1
	$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$ as R_1 and R_3 are identical.	
	Hence the equation of plane	
	$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$	
	$\Rightarrow \begin{vmatrix} x-1 & y-3 & z \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$	1
	$\Rightarrow$ (x-1)(4-2)-(y-3)(2+3)+ z(-4 – 12) = 0	
	$\Rightarrow$ 2(x-1) – 5(y-3) – 16z = 0	
	$\Rightarrow$ 2x – 5y -16z +2 +15 =0	
	$\Rightarrow$ 2x – 5y – 16z= - 17	
	$\Rightarrow \vec{r} \cdot (2\hat{\imath} - 5\hat{\jmath} - 16\hat{k}) = -17$.	1

22.	. xi p(xi) xi p(xi) x_i^2p(xi)	1

xi	p(xi)	xi p(xi)	x_i^2p(xi)
0	1/6	0	0
1	1/2	1/2	½
2	3/10	6/10	6/5
3	1/30	3/30	3/10
Total		12/10	2

E(x)= Mean = Σxip(xi) = 6/5 (1)

Variance(x)= Σpix_i^2- $(\Sigma pixi)^2$ = 2-$(6/5)^2$ = 14/25 (1)

Marks: 1, 1, 1, 1

23\.

Let E1,E2 and A are the event defined below

E1 = the missing card in a heart card

E2 = the missing card is not heart

A = drawing two heart card from the remaining cards (1)

P(E1) =1/4 P(E)= 3/4

P(A/E1)= $\frac{12c_2}{51c_2}$ P(A/E2)= $\frac{13c_2}{51c_2}$ (1)

By Bayes theorem

P(E1/A)= $\frac{P(E1).P(A/E1)}{P(E1).P\left(\frac{A}{E1}\right).P(E2).P(A/E2)}$ (1)

$\frac{\frac{1}{4}\times\frac{12c_2}{51c_2}}{\frac{1}{4}\times\frac{12c_2}{51c_2}+\frac{3}{4}\cdot\frac{13c_2}{51c_2}}$ = 11/50 (1)

SECTION – D

24\.

R= { (a,b): a,b ∈ R and a – b + $\sqrt{3}$ ∈ S

S = Irrational

Reflexive :

Let a ∈ R so a – a +$\sqrt{3}$ ∈ S (2)

Hence, ∀ a ∈ R (a,a) ∈ R so R is reflexive

Symmetric :

Let a = $\sqrt{3}$ b = 2

So, a – b + $\sqrt{3}$ = $\sqrt{3}$ -2 + $\sqrt{3}$ = 2$\sqrt{3}$ -2 ∈ S, so (a,b) ∈ S

Now, b – a + $\sqrt{3}$ = 2 + $\sqrt{3}$ - $\sqrt{3}$ = 2 ∉ S .so, (b,a) ∉ R

So, (a,b) ∈ R but (b,a) ∉ R so, R is not symmetry (2)

Transitive:

Let a = $\sqrt{3}$, b = 2 c = 2$\sqrt{3}$

So, b - a +$\sqrt{3}$ = $\sqrt{3}$ – 2 + $\sqrt{3}$ = 2$\sqrt{3}$ -2 ∈ S (2)

So, (a,b) ∈ R

Now, b – c +$\sqrt{3}$ = 2 – 2$\sqrt{3}$ + $\sqrt{3}$ =2 – $\sqrt{3}$ ∈ S

(b,c) ∈ R

But, a – c + $\sqrt{3}$ = $\sqrt{3}$ – 2$\sqrt{3}$ + $\sqrt{3}$ = 0 ∉ S

So, (a,c) ∉ R hence, R is not a transitive

NO-24. (OR)

$f(x) = 4x^2 + 12x + 15$

<u>One- One</u> :

For any x1 and x2 $\in N$ we find that f(x1) = f(x2)

$4x_1 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

⇨ (x1 – x2) = 0

⇨ X1 = x2 so, f : N -> Range (f) is one to one

Since f : N -> Range (f) so, codomain = Range 2

Hence f is onto , hence f : N -> range (f) is invertible

Let f^{-1} denotes the inverse of f then

f o $f^{-1}(x) = x$ for all $x \in$ Range (f),

$f(f^{-1}(x)) = x$

$4\{f^{-1}(x)\}^2 + 12 f^{-1}(x) + 15 = x$

$= f^{-1}(x) = \frac{-12 \pm \sqrt{144-16(15-x)}}{8}$

$= f^{-1}(x) = \frac{-3+\sqrt{x-6}}{2}$ [$\because f^{-1}(x) \in N$ 2

➔ $f^{-1}(x) > 0$]

2

25.

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2+4+0 & 2-2 & -4+4 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$ 2

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

AB = 6I

$Ax(\frac{1}{6}B) = I$

$A^{-1} = 1/6(B) = 1/6 \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

2

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

AX= C

$X = A^{-1} C$

$X = 1/6 \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$X = 1/6 \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$ 2

$= 1/6 \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

X = 2, y= -1, Z = 4

OR

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

⇨ $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_2 \to R_2 + R_1$

2

⇨ $\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} A \quad R_2 \to R_2 + 2R_3$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 + 2R_2$

⇨ $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -4 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$

2

⇨ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$

2

⇨ Hence $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

26.

$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$

$= \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$

$= \pi \int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} - I$

2

$2I = \pi \int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$2I/2 = 2\pi/2 \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$I = \pi \int_0^{\infty} \frac{dt}{a^2 + b^2 + 2}$ where t = tan x

$I = \frac{\pi}{b^2} \int_0^{\infty} \frac{dt}{(\frac{a}{b})^2 + t^2}$

$I = \frac{\pi}{b^2} \{b/a \tan^{-1}[\frac{bt}{a}]_0^{\infty} \}$

2

$= \frac{\pi}{b^2}$ { b/a x [π/2 – 0]} 2

$= \frac{\pi^2}{2ab}$.

OR

$\int_0^1 e^{2-3x}\,dx$

$I = \int_0^1 e^{2-3x}\ dx$ 2

$= \lim_{h\to 0} h\{ f(0) + f(h) + f(2h) \dots + f(n-1)h\}$

$= \lim_{h\to 0} h[\, e^2 + e^{2-3h} + \cdots e^{2-3(n-1)} h]$

$= \lim_{h\to 0} he^2[\, 1 + e^{-3h} + \cdots + e^{-3(n-1)} h]$

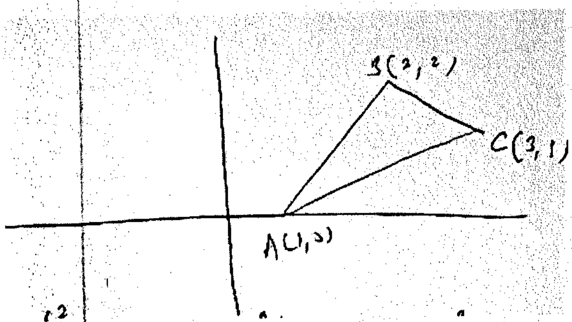
$= \lim_{h\to 0} he^2\ [\frac{(e^{-3nh}-1}{e^{-3h}-1}]$

$= \lim_{h\to 0} \frac{e^{-3}-1}{\frac{e^{-3h}-1}{-3h}}\ X\ \left(-\frac{1}{3}\right) = e^2(e^{-3}-1)\ X\ (-\frac{1}{3})$ 2

$= \frac{1}{3}(e^2 - e^{-1})$

2

27. 2



Area of triangle ABC = $\int_1^2 2(x-1)dx + \int_2^3 (4-x)dx - \int_1^3 \frac{x-1}{2}dx$ 2

$= \quad 2[\frac{x^2}{2} - x]_1^2 + [4x - \frac{x^2}{2}]_2^3 - \frac{1}{2}[[\frac{x^2}{2} - x]_1^3$

= 3/2 2

28. The two Given Planes are

$\vec{r}\cdot(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0$ --(1) 2

$\vec{r}\cdot(2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ --(2)

A plane which contains the line of intersection of plane (1) and (2) is

$\vec{r}\cdot(\hat{\imath} + \hat{\jmath} + 3\hat{k}) - 4 + \lambda\{\vec{r}\cdot(2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5\}$

$\vec{r}\cdot[(1+2\lambda)\hat{\imath} + (2+\lambda)\hat{\jmath} + (3-\lambda)\hat{k}] - 4 + 5\lambda = 0$ --(3)

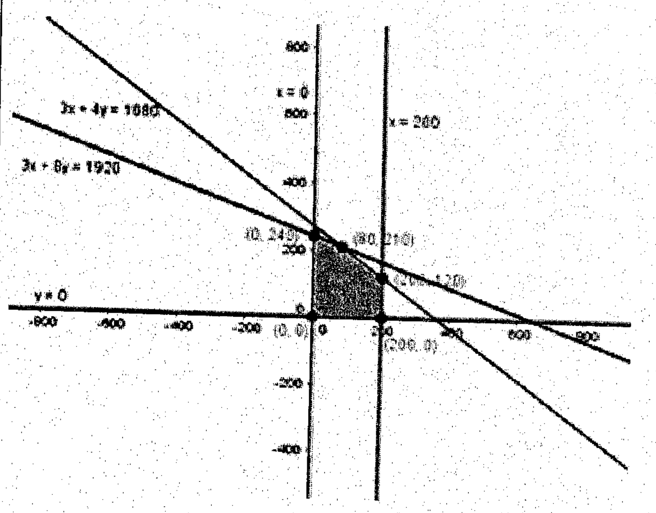
Now the plane (3) is perpendicular to the plane

$\vec{r}\cdot(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$ ----(4)

⇨ (1+2λ)5 + (2+λ) 3 + (3 – λ) (-6) =3 2

⇨ λ= 7/19

	Putting the value of λ in (3) $\vec{r}\cdot(33\hat{\imath}+45\hat{\jmath}+50\hat{k})=41$ This is the required plane.		2
29.	Let x = the number of units of Product 1 to be produced daily y = the number of units of Product 2 to be produced daily To maximize P = (9 – 1.2)x + (8 – 0.9)y = 7.8x + 7.1y subject to the constraints: $\frac{x}{4}+\frac{y}{3}\le 90, or\ 3x+4y\le 1080, \frac{x}{8}+\frac{y}{3}\le 80, or\ 3x+8y\le 1920, x\le 200, x\ge 0, y\ge 0.$		



At the point	P
(0, 0)	0
(200, 120)	2412
(0, 240)	1704
(200, 0)	1560
(80, 210)	2115

The maximum profit = Rs. 2412.

MATHEMATICS

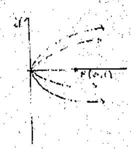
Class XII

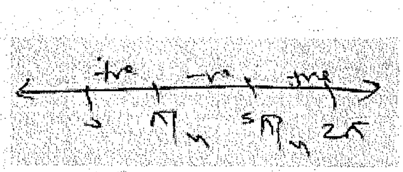
COMMON PRE-BOARD EXAMINATION 2017-2018(SET-3)

MARKING SCHEME

no	ANSWER	MARK(S)
	Section A	
1.	$\|A\|=5, so\ \|6A\|=6^3\ \|A\|= 216 \times 5 = 1080$	1
2.	$y = e^{2\log(3x)}$ $y = 9x^2$ $\frac{dy}{dx} = 18$	1
3.	$\|\vec{a}\| = 1\ ,\ \ \|\vec{b}\| = 2\ ,\ \ \vec{a}\cdot\vec{b} = 1$ $\vec{a}\cdot\vec{b} = \|\vec{a}\|\ \|\vec{b}\|\cos\theta$ $1 = 1 \times 2\cos\theta$ $\text{Cos}\theta = \frac{1}{2}$ $\theta = 60^0$	1
4.	$\frac{d}{dx}[(\frac{dy}{dx})^3] = 3(\frac{dy}{dx})^2 \times (\frac{d^2y}{dx^2})$ Order = 2, degree = 1 Sum = 2 + 1 =3	1
	Section B	
5.	$sin^{-1}\frac{3}{5} + sin^{-1}\frac{8}{17}$ $= tan^{-1}\frac{3}{4} + tan^{-1}\ 8/15$ $= tan^{-1}\left(\frac{\frac{3}{4}+\frac{8}{15}}{1-\frac{3}{4}\times\frac{8}{15}}\right)$	½ ½

	$= \tan^{-1}\left(\frac{45+32}{60-24}\right)$ $= \tan^{-1}\left(\frac{77}{36}\right)$ $= \cos^{-1}\left(\frac{36}{85}\right)$	1
6.	$4\sin^{-1}x + \cos^{-1}x = \pi$ $4\sin^{-1}x + \cos^{-1}x = 2(\sin^{-1}x) + 2\cos^{-1}x$ $2\sin^{-1}x = \cos^{-1}x$ $2\sin^{-1}x = \pi/2 - \sin^{-1}x$ $3\sin^{-1}x = \pi/2$ $\sin^{-1}x = \pi/6$ $x = \sin \pi/6$ $x = \frac{1}{2}$	½ ½ 1
7.	$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\2 & 1\end{bmatrix} = \begin{bmatrix}8 & 5\\20 & 13\end{bmatrix}$ Use first $IA = A$ then $AI = A$ $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -2\end{bmatrix} = \begin{bmatrix}8 & 5\\12 & 8\end{bmatrix}$ $\quad R_2 \to R_2 - R_1$ $\begin{bmatrix}1 & 1\\3 & 1\end{bmatrix}\begin{bmatrix}4 & 3\\-2 & -1\end{bmatrix} = \begin{bmatrix}8 & -3\\12 & -4\end{bmatrix}$ $\quad C_2 \to C_2 - C_1$	1 1
8.	**Y= f(x) =** $x^{\frac{1}{3}}$ Let x= 125, Δx= 2, $y = (125)^{1/3} = 5$ $\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3(125)^{\frac{2}{3}}} = \frac{1}{75}$ dy = $\frac{1}{75} \times 2$ = 2/75	½ ½

	$\Delta y = 0.26$ Therefore $(127)^{\frac{1}{3}} = y + \Delta y = 5 + \Delta y = 5 + 0.26$ $= 5.026$	1
9.	$\mathbf{I} = \int_{-1}^{1} \log(2+3x)/(2-3x)dx$ $f(x) = log(\frac{2+3x}{2-3x})$ $f(-x) = log\left(\frac{2+3x}{2-3x}\right)^{-1}$ $= - log\left(\frac{2+3x}{2-3x}\right)^{-1}$ $= - f(x)$ Hence, f is odd function So, $I = \int_{-1}^{1} \log(2+3x)/(2-3x)dx = 0$	½ ½ 1
10.	$y^2 = 4ax$ -----> (1) $2y\frac{dy}{dx} = 4a$ ------->(2)  From 1 & 2 $y^2 = 2y\,(\frac{dy}{dx})x$ ⇨ $y^2 - 2y\frac{dy}{dx} = 0$ is required D.E	½ ½ 1
11.	$\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$ $\vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a} \perp \vec{c}$ ⇨ $\vec{a} \parallel \vec{b} \times \vec{c}$ ⇨ $\vec{a} = \lambda(\vec{b} \times \vec{c})$--------->(1) ⇨ $1 = \|\lambda\|\|\vec{b} \times \vec{c}\|$ ⇨ $1 = \|\lambda\|\|\vec{b}\|\|\vec{c}\| \sin\frac{\pi}{6}$	½ ½ 1

	⇨ $1 = \frac{1}{2}\|\lambda\|$ ⇨ $2 = \|\lambda\|$ ⇨ $\lambda = \pm 2$ ----> (2) ⇨ *so*, from 1 & 2 $\vec{a} = \pm 2\,(\vec{b} \times \vec{c})$	
12.	P (solved) = p(at least one solved) = 1 – p(none solved) = 1 – p($A' \cap B'$) = 1- P(A') X P(B') $= 1 - \frac{1}{2} \times \frac{2}{3}$ $= 1 - \frac{1}{3} = \frac{2}{3}$	½ ½ 1
	SECTION – C	
13.	dx/dt = ap Cospt, dy/dt = -bp Sinpt dy/dx = $\frac{-bp\ Sinpt}{ap\ Cospt}$ = - b/a tanpt $\frac{d^2y}{dx^2} = \frac{-bp\ sec^2pt}{a}\ X\ dt/dx$ $= \frac{-bp\ sec^2 pt}{a}\ X\ \frac{1}{pa\ cospt} = \frac{-b^2}{(a^2-x^2)y}$ $=(a^2 - x^2)y\frac{d^2y}{dx^2} + b^2 = 0.$	1 1 1 1
14.	(x) = sinx + cosx F'(x) = cosx – sinx Now, f'(x) = 0 gives tanx = 1 => x = $\frac{\pi}{4}$, $\frac{5\pi}{4}$  So, f'(x)> 0 for all x ∈ $[0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$	1 1 1

F'(x)<0 for all x $\in (\frac{\pi}{4}, \frac{5\pi}{4})$ 1

OR

Slope of the tangent to the given curve at 1

Point (x,y) is given by

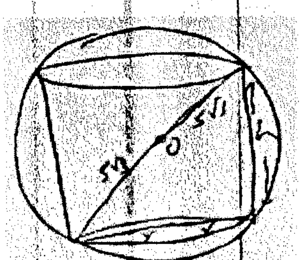
$$\frac{dy}{dx} = \frac{2}{(X-3)^2}$$ 1

⇨ $\frac{2}{(X-3)^2} = 2 \Rightarrow x = 2,4$ 1

⇨ So the points are (2,2) and (4,-2) and the equation

⇨ y-2x+2=0 and y – 2x + 10 = 0 1

15.



$$h^2 + (2r)^2 = \left(10\sqrt{3}\right)^2$$ 1

$$h^2 + 4r^2 = 300$$

$4h^2 = 300 - h^2$ ---------> (i)

Volume of cylinder = $\pi r^2 h$

$v(h) = \pi\left(\frac{3-h^2}{4}\right) \times h$ from equation (i) 1

$V(h) = \frac{\pi}{4}\ [300 - h^3]$

$V'[h] = \ [300 - 3h^2]$

From critical point $v'[h] = 0$,

$3h^2 = 300$,

$h^2 = 100$

h = 10

so, $4r^2 = 300 - 10^2$

	$r^2 = 50$ $r = 5\sqrt{2}$ volume = $\pi r^2 h$ $\pi(50) \times 10 = 500\,\pi$ cubic unit To check maxima $V''[h] = \frac{\pi}{4}[-6h]$ $v''[h] = [-6 \times 10]$ $= -15\,\pi < 0$ So, max volume = $500\,\pi$ cubic unit	1 1
16.	$\cos x = t \Rightarrow -\sin x\,dx = dt$ $I = \int \frac{-dt}{(t^2+1)(t^2+4)}$ Put $t^2 = y$ $\frac{-1}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$ $-1 = A(y+4) + B(y+1)$ $(A+B) = 0$ $4A+B = -1$ ⇨ $A = -1/3,\ B = 1/3$ This given integral = $-1/3 \int \frac{dt}{t^2+1} + 1/3 \int \frac{dt}{t^2+4}$ $= -1/3\, tan^{-1}t + 1/6\, tan^{-2}\left(\frac{t}{2}\right) + C$ $= -1/3\, tan^{-1}(Cos\ x) + 1/6\, tan^{-1}\left(\frac{Cos\ x}{2}\right) + c$	1 1 1 1
17.	1) Applying $C_1 \rightarrow C1 + C_2 + C_3$	

	2) Taking 2 common from $\boldsymbol{C_1}$	1
	3) Applying $\boldsymbol{C_2 \to C_2 - C_1}$	
	4) Applying $C_3 \to C_3 - C_1$	1
	5) Applying $C_1 \to C_1 + C_2 + C_3$	1
	6) Taking (-1) common from C_2 & C_3	1
18.	$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x \rangle 0 \end{cases}$	1
	Since, f(x) is continuous at 0	1
	So, L.H.L. = R.H.L. = f(0)=a	
	L.H.L, $\lim_{x\to 0^-} f(x)$	
	$= \lim_{x \to 0^-} a \sin\{^{\pi}/_{6}\ (x+1)$	
	= a	1
	R.H.L, $\lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3}$	
	$\lim_{x\to 0^+} \frac{\frac{\sin x}{cos x} - \sin x}{x^3}$	1
	$\lim_{x\to 0^+} \frac{sin x\ (\frac{1-cos x}{\cos x})}{x^3}$	
	$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{sin^2\ x/2}{x^2}$	
	$\lim_{x\to 0^+} \frac{\tan x}{x} \times \lim_{x\to 0^+} \frac{2\ sin^2\ x/2}{\frac{x^2}{4} \times 4}$	
	= 1 x 2/4 = 1/2	
	So, a = ½	
	OR	
	since f is differentiable at 1, f is continues at 1.	

Hence, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x + 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(ax^2 + b) = a + b$

f(1) = 3 1

As, f is continues at 1 , we have a + b = 3 ----(1)

L.H.D. $f'(1) = \lim_{x\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{x\to 0} \frac{a(1-h)^2+b-3}{-h}$

$= \lim_{h\to 0} \frac{a+ah^2-2ah+b-3}{-h}$ 1

$= \lim_{h\to 0} -ah + 2a$ (from 1)

= 2a

R.H.D $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{x\to 0} \frac{2(1+h)+1-3}{h}$ 1

=2

As, f is differentiable at 1. We have 2a= 2, i.e. a = 1, b = 2.

19.

$$\frac{dy}{dx} - 3y\cot x = \sin 2x$$

Here P= -3 cot x, Q= sin 2x 1

I.f $= e^{\int p dx} = e^{\int -3\cot x\, dx} = e^{-3\log(\sin x)}$

$= e^{\log\left(\frac{1}{sin^3 x}\right)} = \frac{1}{sin^3 x}$

$y \times I \cdot F = \int Q \cdot I \cdot F\, dx$ 1

$= \quad y \cdot \frac{1}{sin^3 x} = \int sin2x \times \frac{1}{sin^3 x} d$

$= \quad 2\int cotx\ cosecx\ dx + c$ 1

$\frac{y}{sin^3 x}$ = -2 cosecx + c

y = -2 sinx + $c sin^3 x$ 1

2 = -2 + c , C = 4

y = -2 sin^2x +4 $sin^3 x$

OR

$[x\sin^2(\frac{y}{x}) - y]dx + x\,dy = 0$, $\frac{dy}{dx} = \frac{y - x\sin^2(\frac{y}{x})}{x}$ 1

$\frac{dy}{dx} = (\frac{y}{x}) - \sin^2(\frac{y}{x})$, let $\frac{y}{x}$ = v => y = vx

⇨ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

⇨ $-\int \frac{dV}{\sin^2 v} = \int \frac{dx}{x}$ 1

⇨ $\cot(\frac{y}{x}) = \log|x| + c$

⇨ $\cot(\frac{\pi}{4}) = \log(1) + c$, c = 1

⇨ $\cot(\frac{y}{x}) = \log|x| + 1$ 1

⇨ OR

⇨ $\cot(\frac{y}{x}) = \log|xe|$ 1

20. $\vec{d} \perp \vec{a}$ *and* $\vec{d} + \vec{b}$ 1

So, $\vec{d} \parallel \vec{a} \times \vec{b}$

⇨ $\vec{d} = \lambda(\vec{a} \times \vec{b})$ --(1) 1

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$\vec{a} \times \vec{b} = 32\hat{\imath} - \hat{\jmath} + (-14)\hat{k}$ ---(2) 1

From (1) and (2)

$\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$

But $\vec{d} \cdot \vec{c} = 15$ 1

=> 64λ + λ + 56λ = 15

=> λ = 5/3

So, $\vec{d} = \frac{5}{3}(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$

$= \frac{160}{3}\hat{\imath} - \frac{5}{3}\hat{\jmath} - \frac{70}{3}\hat{k}$.

21. $l_1: \frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $l_2: \frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$ 1

If l_1 and l_2 are co-planner then,

$$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_1 - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0 \quad \text{as } R_1 \text{ and } R_3 \text{ are identical.}$$ 1

Hence the equation of plane

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$ 1

$$\Rightarrow \begin{vmatrix} x-1 & y-3 & z \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$$ 1

⇨ (x-1)(4-2)-(y-3)(2+3)+ z(-4 – 12) = 0

⇨ 2(x-1) – 5(y-3) – 16z = 0

⇨ 2x – 5y -16z +2 +15 =0

⇨ 2x – 5y – 16z= - 17

⇨ $\vec{r} \cdot (2\hat{i} - 5\hat{j} - 16\hat{k}) = -17.$

22. Let x denote the random variable and can take value 0,1,2, n=2, P=1/4, q= ¾ 1

x_i	0	1	2	Total	Marks
P_i	${}^2C_0\left(\frac{3}{4}\right)^2 = \frac{9}{16}$	${}^2C_1 \frac{1}{4}\left(\frac{3}{4}\right) = \frac{6}{16}$	${}^2C_2\left(\frac{1}{4}\right)^2 = \frac{1}{16}$		[1+1/2]
$x_i P_i$	0	6/16	2/16	1/2	
x_i^2	0	6/16	4/16	5/8	[1/2]

1

Mean = $\sum xi\, pi = 1/2$

Value = $\sum xi^2\, p(xi) - (\sum xi\, pxi)^2$ 1

= 5/8 – 1/4 = 3/8.

1

23. Let E1,E2 and A are the event defined below

		E1 = the missing card in a heart card E2 = the missing card is not heart A = drawing two heart card from the remaining cards P(E1) =1/4 P(E)= 3/4 P(A/E1)= $\frac{12c_2}{51c_2}$ P(A/E2)= $\frac{13c_2}{51c_2}$ By Bayes theorem P(E1/A)= $\frac{P(E1).P(A/E1)}{P(E1).P\left(\frac{A}{E1}\right).P(E2).P(A/E2)}$ $\frac{\frac{1}{4}\times\frac{12c_2}{51c_2}}{\frac{1}{4}\times\frac{12c_2}{51c_2}+\frac{3}{4}\cdot\frac{13c_2}{51c_2}}$ = 11/50	1 1 1 1
		SECTION – D	
24.		R= { (a,b): a,b ∈ R and a – b + $\sqrt{3}$ ∈ S = Irrational <u>Reflexive :</u> Let a ∈ R so a – a +$\sqrt{3}$ ∈ S Hence, ∀ a ∈ R (a,a) ∈ R so R is reflexive <u>Symmetric :</u> Let a = $\sqrt{3}$ b = 2 So, a – b + $\sqrt{3}$ = $\sqrt{3}$ -2 + $\sqrt{3}$ = 2$\sqrt{3}$ -2 ∈ *S*, so (a,b) ∈ S Now, b – a + $\sqrt{3}$ = 2 + $\sqrt{3}$ - $\sqrt{3}$ = 2 ∉ S .so, (b,a) ∉ R So, (a,b) ∈ R but (b,a) ∉ R so, R is not symmetry Transitive: Let a = $\sqrt{3}$, b = 2 c = 2$\sqrt{3}$	2 2

	So, b - a $+\sqrt{3} = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$ -2 $\in S$ So, (a,b) $\in$ R Now, b – c $+\sqrt{3} = 2 - 2\sqrt{3} + \sqrt{3} = 2 - \sqrt{3} \in S$ (b,c) $\in R$ But, a – c + $\sqrt{3} = \sqrt{3} - 2\sqrt{3} + \sqrt{3} = 0 \notin S$ So, (a,c) $\notin$ R hence, R is not a transitive OR f(x) = $4x^2$ + 12x + 15 <u>One- One</u> : For any x1 and x2 $\in N$ we find that f(x1) = f(x2) $4x_1 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ ⇨ (x1 – x2) = 0 ⇨ X1 = x2 so, f : N -> Range (f) is one to one Since f : N -> Range (f) so, codomain = Range Hence f is onto , hence f : N -> range (f) is invertible Let f^{-1} denotes the inverse of f then f o f^{-1}(x) = x for all x $\in$ Range (f), f(f^{-1}(x)) = x $4\{f^{-1}(x)\}^2 + 12 f^{-1}(x) + 15 = x$ $= f^{-1}(x) = \frac{-12 \pm \sqrt{144-16(15-x)}}{8}$ $= f^{-1}(x) = \frac{-3+\sqrt{x-6}}{2}$ [$\because f^{-1}$ (x) $\in$ N ➔ f^{-1} (x) >0]	2 2 2 2
25.	$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$	

$$AB = \begin{bmatrix} 2+4+0 & 2-2 & -4+4 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$ 2

AB= 6I

$Ax(\frac{1}{6}B) = I$

$$A^{-1} = 1/6(B) = 1/6 \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

AX= C

$X = A^{-1}C$

$$X = 1/6 \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$ 2

$$X = 1/6 \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$$

$$= 1/6 \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$ 2

X = 2, y= -1, Z = 4

OR

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_2 \to R_2 + R_1$$ 2

$$\Rightarrow \begin{bmatrix}1 & 2 & -2\\ 0 & 1 & 0\\ 0 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 2\\ 0 & 0 & 0\end{bmatrix} A \qquad R_2 \rightarrow R_2 + 2R_3$$

2

$R_1 \text{->} R_1 + 2R_2$, $R_3 \text{->} R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix}1 & 0 & -2\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-1 & -2 & -4\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix} A$$

$$\Rightarrow \text{Hence } A^{-1} = \begin{bmatrix}3 & 2 & 6\\ 1 & 1 & 2\\ 2 & 2 & 5\end{bmatrix}$$

2

26.

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$= \int_0^{\pi} \frac{(\pi - x)dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$$

$$= \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \quad -I$$

2

$$2I = \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$2I/2 = 2\pi/2\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

2

$$I = \pi \int_0^{\propto} \frac{dt}{a^2 + b^2 + 2} \qquad \text{where } t = \tan x$$

$$I = \frac{\pi}{b^2}\int_0^{\propto} \frac{dt}{(\frac{a}{b})^2 + t^2}$$

2

$$I = \frac{\pi}{b^2}\{b/a \tan^{-1}[\frac{bt}{a}]_0^{\propto}\}$$

$$= \frac{\pi}{b^2}\{b/a \times [\pi/2 - 0]\}$$

$$= \frac{\pi^2}{2ab}.$$

OR

$\int_0^1 e^{2-3x}\,dx$ 2

$I = \int_0^1 e^{2-3x}\,dx$

$= \lim_{h\to 0} h\{ f(0) + f(h) + f(2h) \ldots + f(n-1)h\}$

$= \lim_{h\to 0} h[\, e^{2} + e^{2-3h} + \cdots e^{2-3(n-1)}h]$ 2

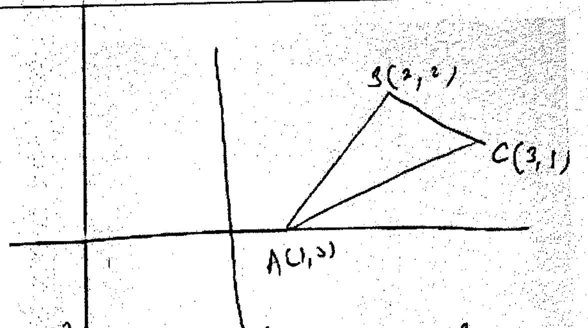
$= \lim_{h\to 0} he^{2}[\,1 + e^{-3h} + \cdots + e^{-3(n-1)}h]$

$= \lim_{h\to 0} he^{2}\ [\frac{(e^{-3nh}-1}{e^{-3h}-1}]$

$= \lim_{h\to 0} \frac{e^{-3}-1}{\frac{e^{-3h}-1}{-3h}}\ X\left(-\frac{1}{3}\right) = e^{2}(e^{-3}-1)\ X\left(-\frac{1}{3}\right)$ 2

$= \frac{1}{3}(e^{2} - e^{-1})$

27. 2



2

Area of triangle ABC $= \int_1^2 2(x-1)dx + \int_2^3 (4-x)dx - \int_1^3 \frac{x-1}{2}dx$ 2

$= 2[\frac{x^2}{2} - x]_1^2 + [4x - \frac{x^2}{2}]_2^3 - \frac{1}{2}[[\frac{x^2}{2} - x]_1^3$

$= 3/2$

28. The two Given Planes are

$\vec{r}\cdot(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0$ --(1)

$\vec{r}\cdot(2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ --(2) 2

A plane which contains the line of intersection of plane (1) and (2) is

$$\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+3\hat{k})-4+\lambda\{\vec{r}\cdot(2\hat{\imath}+\hat{\jmath}-\hat{k})+5\}$$

$$\vec{r}\cdot[(1+2\lambda)\hat{\imath}+(2+\lambda)\hat{\jmath}+(3-\lambda)\hat{k}]-4+5\lambda=0 \quad \text{--(3)}$$

Now the plane (3) is perpendicular to the plane

$$\vec{r}\cdot(5\hat{\imath}+3\hat{\jmath}-6\hat{k})+8=0 \qquad \text{----(4)}$$ 2

⇨ $(1+2\lambda)5 + (2+\lambda)\,3 + (3-\lambda)\,(-6) = 3$

⇨ $\lambda = 7/19$ 2

Putting the value of λ in (3)

$\vec{r}\cdot(33\hat{\imath}+45\hat{\jmath}+50\hat{k}) = 41$ (This is the required plane)

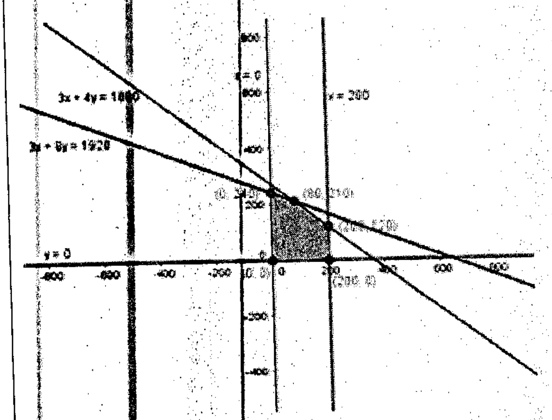
29. Let x = the number of units of Product 1 to be produced daily

y = the number of units of Product 2 to be produced daily

To maximize P = (9 – 1.2)x + (8 – 0.9)y = 7.8x + 7.1y

subject to the constraints:

$$\frac{x}{4}+\frac{y}{3}\le 90, or\ 3x+4y\le 1080, \frac{x}{8}+\frac{y}{3}\le 80, or\ 3x+8y\le 1920, x\le 200, x\ge 0, y\ge 0.$$



At the point		P
(0, 0)		0
(200, 120)		2412
(0, 240)		1704
(200, 0)		1560
(80, 210)		2115

The maximum profit = Rs. 2412. 2